微積分(II) 期末模擬考試 (2002/6/18)

Problem 1. (page 641)
In (a), (b) set up integrals for the volume with the given boundaries.
(a) $z = \cos(x^2 + y^2)$, $(z = 0; \frac{3\pi}{2} \leq x^2 + y^2 \leq \frac{5\pi}{2})$
(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Problem 2. (page 642) Prove that $\frac{d^n}{dt^n} \int_{-t}^{t} f(x+t)dx = 2^n f^{[n-1]}(2t)$, $(n \geq 1)$.

Problem 3. (page 641) Show that $\int_0^1 \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2} dydx \neq \int_0^1 \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2} dxdy$

Problem 4. (page 639) Convert the following integral to polar coordinates and evaluate. $\int_0^1 \int_{x^2}^{x} (x^2+y^2)^{-1/2} dydx$.

Problem 5. (page 639) Let R be the triangular region in the xy plane bounded by $x = 0, y = 0$, and $x + y = 1$. Compute the double integral $\int_R \exp[(x-y)/(x+y)]$.

Problem 6. (page 640)
(a) Compute the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$,
(b) Compute triple integral $\int\int\int xyz$ over the ellipsoid in (a) above.

Problem 7. (page 628) Use Green's theorem to evaluate $\oint_\lambda [\frac{1}{y}dx + \frac{1}{x}dy]$ where λ is the closed curve formed by $y = 1, x = 4$, and $y = \sqrt{x}$.

Problem 8. (page 615) Find the volume of the following regions of space, and also find the value of the triple integral of the given function f over each region
(a) Region S bounded by the cylinder $x^2 + y^2 = 16$ and the planes $z = 0$, and $z = 3$; $f(x, y, z) = xz + yz$.
(b) Region S bounded by the cylinder $x^2 = z$ and $x^2 = 4 - z$, and the planes $y = 0$, and $z + 2y = 4$; $f(x, y, z) = 2x - z$.

Problem 9 (page 601) In polar coordinate system, find the volume of the solid bounded by the cone $z = 2r$, $(r \geq 0)$, the cylinder $r = 1 - \cos\theta$, and the plane $z = 0$.

Problem 10. (page 602) Show that $\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$.

Problem 11. (page 597)
(a) Find the volume V of the solid bounded by the graph of the equation $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$.
(b) Use repeated integrals to find the volume of the solid common to two right circular cylinders of radius r whose axes intersect orthogonally.
(c) Find the volume of the solid under the plane $z = 3x + y$ and above the part of the ellipse $4x^2 + 9y^2 \leq 36$ in the first quadrant. Sketch

Problem 11. (page 585) Show that $\int_0^1 \int_0^1 f(x, y)dydx = -\int_0^1 \int_0^1 f(x, y)dxdy = \frac{1}{2}$, where $f(x, y) = (x - y)/(x + y)^3$.

Problem 12. (page 576) Evaluate the line integral $\int_\lambda x^2dy + y^2dz + z^2dx$ along the curve $\lambda(t) = (t^2, t+1, t-1)$, domain $\lambda = [0, 1]$.

Problem 13. (page 576) If $F(x, y) = \frac{xy(x^2-y^2)}{x^2+y^2}$, $(x, y) \neq (0, 0)$, $F(0, 0) = 0$, show that $F_{xy}(0, 0) \neq F_{yx}(0, 0)$.

Problem 14. (page 574) Evaluate the line integral $\int_\lambda [(x+y)dx + (x-y)dy]$ if λ is the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ traversed in a counterclockwise direction.

Problem 15. (page 570) Determine which point of the sphere $x^2 + y^2 + z^2 = 25/2$ is at the greatest distance from the point $(3, 4, 5)$.

Problem 16. (page 575) Find the maximum directional derivative of the function $F(x, y) = 100 - 2x^2 - y^2$ at the point $(5, 5)$.

Problem 17. (page 569) Find the extrema of the following function.

(a) $F(x, y) = x^2 + \frac{2}{xy^2} + y^2$.

(b) $g(x, y) = \sin x + \sin y + \sin(x + y)$.

Problem 18. (page 569) Find the minimum distance between the point $(1, -1, 2)$ and the plane $3x + y - 2z = 4$.

Problem 19. (page 570) Determine the values of a and b such that the ellipse $x^2/a^2 + y^2/b^2 = 1$ has least area and contains the circle $(x-1)^2 + y^2 = 1$.

Problem 20. (page 564) Suppose $y = f(x)$ satisfies the equation $\arctan(\frac{x}{y}) + y^3 - 1 = 0$, find $\frac{dy}{dx}$.

Problem 21. (page 562)

(a) If $z = f(u/v)/v$, show that $v(\partial z/\partial v) + u(\partial z/\partial u) + z = 0$.

(b) If $F(x, y) = f(y + ax) + g(y - ax)$, show that $\frac{\partial^2 F}{\partial x^2} = a^2 \frac{\partial^2 F}{\partial y^2}$.

Problem 22. (page 556) Find the tangent plane to the graph of the equation $x^2 + y^2 - 4z^2 = 4$ at the point $(2, -2, 1)$.

Problem 22. (page 544) Find the maximum directional derivative of the function f defined by $f(x, y, z) = x^2 + y^2 + z^2$ at the point (a, b, c).

Problem 23. (page 541) Discuss the continuity of the function $G(x, y) = \frac{x^3+y^3}{x^2+y^2}$ if $(x, y) \neq (0, 0)$; $G(0, 0) = 0$.

Problem 24. (page 507) If $u = \langle a_1, a_2, a_3 \rangle$, $v = \langle b_1, b_2, b_3 \rangle$ and $w = \langle c_1, c_2, c_3 \rangle$, then show that

$$u \bullet (v \times w) = (u \times v) \bullet w = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Give a geometric interpretation of this number.

Problem 25. (page 522) Find the length of the space curve defined by $\lambda(t) = (t^2 + 1, t^2 - 1, 8t)$, domain $\lambda = [1, 3]$.

Problem 26. (page 527) Find the maximum and minimum radii of curvature for the curve $\lambda(t) = (a \cos t, b \sin t)$.

Problem 27. (page 526) Reparametrize by arc length the curve λ defined by $\lambda(t) = (t \sin t, t \cos t, \frac{2\sqrt{2}}{2} t^{3/2})$.

Problem 28. (page 513) Find the equation of the paraboloid of revolution obtained by rotating the parabola $y^2 = 4px$ in the xy plane about the x axis. Sketch.

Problem 29. (page 500) Find an equation of the plane passing through the three points $(-1, 1, 2)$, $(2, 0, -3)$, and $(5, 1, -2)$.

Problem 30. (page 497) Find the direction cosines and direction angles of the half-line emanating from the origin and passing through the point $(1, 1, \sqrt{2})$.

Problem 31. (page 480) Find the surface area of a sphere of radius r.

Problem 32. (page 480) One loop of the lemniscate $r^2 = cos2\theta$ is rotated about the polar axis. Find the area of the surface generated.

Problem 33. (page 474) Exercise 7.

Problem 34. (page 473) Exercises 5 and 9.

Problem 35. (page 467) Exercises Part(I) 5, 7, 15, 17, 19, 22, Part(II) 3.

Problem 36. (page 462) Exercises Part(I) 9, 10, 11, 23. Part(II) 3.

Problem 37. (page 451) Exercise Part(I) 1.

Problem 38. (page 434) Exercises Part(I) 24, Part(II) 3, 4.