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Calculus Test

2005/12/14 (P1)

(1) (10%) The radius of a ball is measured to be 0.7 inch. If the measurement is correct to within 0.01 inch, estimate the propagated error in the volume V of the ball bearing.

(2) (20%) The range R of a projectile is

$$R = \frac{V_0^2}{32}(\sin 2\theta),$$

where V_0 is the initial velocity in feet per second and θ is the angle of elevation. If $V_0 = 2200$ feet per second and θ is changed from $10°$ to $11°$, use differentials to approximate the change in the range.

(3) (15%) Use Newton's Method to approximate the zeros of

$$f(x) = 2x^3 + x^2 - x + 1.$$

Continue the iterations until two successive approximations differ by less than 0.01.

(4) (20%) Find the following (indefinite) integral respectively.

(a) $\int (\sqrt[4]{x^3} + 1)\,dx$ (b) $\int \frac{\sin x}{1-\sin^2 x}\,dx$

(c) $\int \frac{x^2+2x-3}{x^4}\,dx$ (d) $\int_{-1}^{1} (|2x-1| + |2x+1|)\,dx$

(5) (20%) (a) Find the following formula respectively

(i) $\sum_{i=1}^{n} i^2$ (ii) $\sum_{i=1}^{n} i^3$ (iii) $\sum_{i=1}^{n} i^4$

(b) Using above results (a) to find

$$\lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3 \frac{2}{n}\right] \text{ and } \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\frac{i}{n}\right)^4 \cdot \frac{1}{n}\right]$$

(6) Determine whether the following statement is true or
(15%) false. If it is false, explain why or give an example that shows it is false.

(a) $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

(b) $\int_a^b f(x)g(x)\,dx = \left[\int_a^b f(x)\,dx\right]\left[\int_a^b g(x)\,dx\right]$