

A.4

## Final Exam of Calculus I (Fall 2000)

**Problem 1. (20pts)**

(a) Show that  $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$ .

(b) Find  $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$ .

**Problem 2. (10pts)**Is the function  $f(x) = \lim_{n \rightarrow \infty} \frac{2x^{2n} + 2x^3 + 1}{2x^{2n+1} + 1}$  continuous at  $x = -1$ ?**Problem 3. (10pts)** Let  $f(x) = \frac{x(2+x)(5+x)(9-x)}{(1-x)(3+x)^2(6-x)}$ . Find  $f'(0)$ .**Problem 4. (10pts)** Find the slope of the tangent line to the graph  $y = x^3$  at  $x = 0$ . (show your work)**Problem 5. (10pts)**(a) Show that  $g(x) = |x|$  is not differentiable at  $x = 0$ .(b) Find a formula for  $g'(x)$ .**Problem 6. (25pts)**(a) Let  $y = (x^3 + x - 2)^{10}$ . Find  $y'$ .(b) Prove that  $\frac{d \sin x}{dx} = \cos x$ .(c) Show that  $\frac{d \csc x}{dx} = -\csc x \cot x$ .(d) Let  $f(x) = \cos(\sqrt{1 + \sin x})$ . Find  $f'(x)$ .(e) Let  $f(x) = \frac{x^2 - x^3}{1+x}$ . Find  $f'(x)$ .**Problem 7. (10pts)** Let  $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ .(a) Determine the intervals on which  $f(x)$  is increasing or decreasing.(b) Determine the open intervals on which  $f(x)$  is concave upward and open intervals on which  $f(x)$  is concave downward.**Problem 8. (20pts)** Find the relative extrema of the following functions, if any(a)  $f(x) = \frac{x}{2} - \sin x$ .(b)  $f(x) = |\sin x|$ .