

Midterm Test of Calculus II (Spring 2001)

Problem 1. (10pts)

Suppose f and g are inverse functions of each other which are differentiable. Show that $g'(x) = \frac{1}{f'(g(x))}$ whenever $f'(g(x)) \neq 0$.

Problem 2. (20pts) Find $\frac{d \tan^{-1} x}{dx}$.

Problem 3. (10pts) Find $\frac{d \tan^{-1} \frac{1}{x}}{dx}$.

Problem 4. (10pts)

Show that $\tan^{-1} \frac{1}{x} = -\tan^{-1} x + \frac{\pi}{2}$ for every $x \neq 0$

Problem 5. (20pts)

(a) State the *Roll's Theorem*

(b) State the *Mean Value Theorem*.

(c) State the *Taylor's Theorem*.

(d) State the *L'Hôpital's first rule and second rule*.

Problem 6. (10pts) Estimate the value $\sqrt{24}$ using mean value theorem.

Problem 7. (20pts) If $y = x^{x^x}$, $x > 0$. Find $\frac{dy}{dx}$.

Problem 8. (10pts)

Suppose $x^y = y^{2x}$ ($x > 0, y > 0$), find $\frac{dy}{dx}$.

Problem 9. (10pts) Find $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{\cot x}{x} \right)$.

Problem 10. (10pts) Find $\int \frac{1}{e^x+1} dx$.

Problem 10. (10pts) Find $\int \frac{1}{3x^2-4x-4} dx$.