DIFFERENTIATION

Problems:

1. Show that if f is differentiable at a then f is continuous at a.

2. Give an example of a function f which is continuous at a point a but not differentiable at a.

3. f(x,y) $= \begin{cases} ax^2+bx+7 & \text{if } x<1 \\ 2bx+2a & \text{if } x\geq 1 \end{cases}$
   Suppose that f is differentiable at 1
   Find a, b.

4. Find $\frac{dy}{dx}$ for the following:
   (1) $y=\sin^3(\tan x)$
   (2) $y=\sec(\cos^{\frac{1}{2}}(x^2+5-\frac{1}{x}))$
   (3) $y=x^2\cot(\csc x)+x\sin x\cos x$
   (4) $y=\sin(x^2+1)+\frac{x+2}{x^3+1}$

5. $y=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_3x^3+a_2x^2+a_1x+a_0$
   Find $y',\ y'',\ y^{(n)}$ *and* $y^{(n+1)}$.

6. Find $\frac{dy}{dx}$ if $y^3=x^2+\cos\frac{x}{y^2}$.

7. Find $\frac{d^2y}{dx^2}$ if $xy=\sin x+\cos y$.

8. Let C denote the curve $y^4-4y^2=x^4-9x^2$.
   (1) Show that (3,2) and (3,-2) are points on the curve C.
   (2) Find the equation for the tangent to the curve C at (3,2).
   (3) Find the equation for the normal to the curve C at (3,-2).

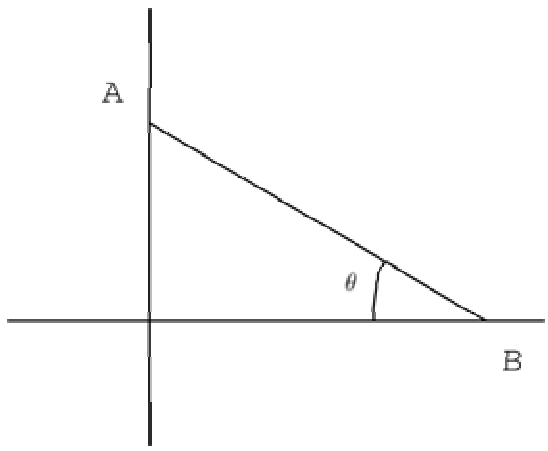
9. Let C be the curve described by

$$x=t-\sin t,\quad y=1-\cos t$$

   (1) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ for t=$\frac{\pi}{3}$.

(2) Find the equation for the line tangent to the curve C at the point defined by t=$\frac{\pi}{3}$.

10. A, B are walking on the streets that meet at right angles. A approaches the intersection at 3 m/sec; B moves away from the intersection at 2 m/sec.
At what rate is the angle θ changing and at what rate is the distance between A and B changing when A is 10 m from the intersection and B is 25 m from the intersection?



11. (1) $f(x) = \sqrt[3]{x}$
Find the differential at f.
(2) Use the differential to estimate $\sqrt[3]{1000.2}$.